2018 Workshop on Topological Methods in Analysis and Algebra
Borriana, June 14th-15th, 2018

On the poset of dense pseudocompact subgroups of a compact group

by
Dikran Dikranjan
Dipartimento di Matematica e Informatica
Università di Udine
Italy

Abstract

It is well known that every compact group of uncountable weight contains proper dense pseudocompact subgroups ([1, 2, 3]). Wilcox [5] showed that not every compact abelian group of uncountable weight contains a pair of dense pseudocompact subgroups with trivial intersection.

Given a compact abelian group K, we study the poset $\mathcal{P}(K)$ of dense pseudocompact subgroups of K and the subgroup $\mathbf{den}_{psc}(K) := \bigcap \mathcal{P}(K)$. It turns out that for every compact abelian group K:

(a) the subgroup $\mathbf{den}_{psc}(K)(K)$ is compact metrizable and coincides with the intersection of just two members $D_0, D_1 \in \mathcal{P}(K)$;

(b) if $\mathbf{den}_{psc}(K) = \{0\}$, then there exists some independent family $\mathcal{F}$ in $\mathcal{P}(K)$ such that:

  (b_1) $|\mathcal{F}| = r(K)$ (the free rank of K), if mK is not metrizable for any positive $m \in \mathbb{N}$; otherwise,

  (b_2) when K is (necessarily) bounded torsion, $|\mathcal{F}|$ coincides with the least leading Ulm-Kaplansky invariant of K.

(c) in option (b_1) the members of $\mathcal{F}$ can be chosen to be free subgroups of K precisely when K admits a free dense subgroup (equivalently, when $r(K) \geq d(K)$); $\mathcal{F}$ can have (the maximum possible) size $|K|$ if and only if $r(K) = |K|$;

(d) in option (b_2) the family $\mathcal{F}$ can have size $|K|$ if and only if all leading Ulm-Kaplansky invariants of K coincide with $|K|$.

The essential part of these results were obtained jointly with W. Comfort shortly before he passed away. They generalise known facts obtained by Wilcox [5] and by Itzkowitz and Shakhmatov [4]

References

[1] W. W. Comfort and Lewis C. Robertson, *Proper pseudocompact extensions of compact Abelian group topologies*, Proc. Amer. Math. Soc **86** (1982), 173–178.

[2] W. W. Comfort and T. Soundararajan, *Pseudocompact group topologies and totally dense subgroups*, Pacific J. Math. **100** (1982), 61–84.

[3] G. L. Itzkowitz, *Extensions of Haar measure for compact connected Abelian groups*, Indag. Math. 27 (1965), 190–207.

[4] G. Itzkowitz and D. Shakhmatov, *Large families of dense pseudocompact subgroups of compact groups*, Fund. Math. **147** (1995), no. 3, 197–212.

[5] H. J. Wilcox, *Pseudocompact groups*, Pacific J. Math. **19** (1966), 365–37