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A version of the Stone-Weierstrass theorem in fuzzy analysis

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Abstract

Fuzzy numbers provide formalized tools to deal with non-precise quantities. They are indeed fuzzy sets in the real line and were introduced in 1978 by Dubois and Prade, who also defined their basic operations. Since then, Fuzzy Analysis has developed based on the notion of fuzzy number just as much as classical Real Analysis did based on the concept of real number. Such development was eased by a characterization of fuzzy numbers provided in 1986 by Goetschel and Voxman leaning on their level sets.

As real-valued functions do in the classical setting, fuzzy-number-valued functions, that is, functions defined on a topological space taking values in the space of fuzzy numbers, play a central role in Fuzzy Analysis. Namely, fuzzy-number-valued functions have become the main tool in several fuzzy contexts, such as fuzzy differential equations, fuzzy integrals or fuzzy optimization. However the main difficulty of dealing with these functions is the fact that the space they form is not a linear space; indeed it is not a group with respect to addition.

We focus on the conditions under which continuous (with respect to the supremum metric) fuzzy-number-valued functions defined on a compact Hausdorff space can be (uniformly) approximated to any degree of accuracy. More precisely and based on ideas of R.I. Jewett and J.B. Prolla, we provide a sufficient set of conditions on a subspace of the space of fuzzy-number-valued functions in order that it be dense, which is to say a Stone-Weierstrass type result. The celebrated Stone-Weierstrass theorem is one of the most important results in classical Analysis, plays a key role in the development of General Approximation Theory and, particularly, is in the essence of the approximation capabilities of neural networks. We also obtain a similar result for interpolating families of continuous fuzzy-number-valued functions in the sense that the uniform approximation can also demand exact agreement at any finite number of points.