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The Uniformization Theorem: Fuchsian groups and algebraic curves

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Abstract

The Uniformization Theorem for Riemann surfaces generalises the well known Riemann Mapping Theorem, going from classifying the simply connected domains of the complex plane, to classify all the simply connected Riemann surfaces.

The Uniformization Theorem states that the only three simply connected Riemann surfaces are the sphere, the complex plane and the Poincar's disc (or the half plane). The main objective of this talk is to apply this theorem to classify part of the other Riemann surfaces. In order to do so, we will study some basics of Kleinian and Fuchsian groups. This will allow us to determine all Riemann surfaces with abelian fundamental group.

Finally, if time allows, we will use this theorem to classify algebraic curves with the intention of uniformise all the projective algebraic planar curves of genus 1, it is still an open problem for curves with bigger genus.