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Strong norming sets and the separable quotient problem
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Abstract

Mazur's open problem asks if an infinite dimensional Banach space E admits an infinite dimensional separable quotient (1932 and SQP in brief). Saxon-Wilansky's paper [6] contains six equivalent properties to SQP and the property P2, *the existence of a dense non-barrelled subspace*, is due to Saxon and Wilansky.

A subset C of a normed space $(E, \|\cdot\|)$ is *norming* if its bipolar $C^{oo} = \overline{\text{abcx}\, C}$ is a bounded neighborhood of 0. C is *strong norming* if each increasing covering of C contains a norming set.

We will present some examples and topological properties of strong norming sets that enables to get that an infinite dimensional Banach space $(E, \|\cdot\|)$ has SQP if and only if the unit sphere S_E contains a dense non strong norming subset.

Finally, we will survey some general results and problems related to SQP following [1], [3], [4], [5] as well as some particular Banach spaces of vector-valued functions, linear operators and vector measures with infinite dimensional separable considered in [2].

References

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