Free and universal objects in topological algebra, and more
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Abstract

I'll present a few results of mine, some old and some not so old.

1. Two Tikhonov spaces X and Y are said to be L-equivalent if the free LCS (locally convex spaces) $L(X)$ and $L(Y)$ are isomorphic. The spaces X and Y are l-equivalent if $L(X)$ and $L(Y)$ become isomorphic if equipped with the weak topology (equivalently, if the function spaces $C_p(X)$ and $C_p(Y)$ are isomorphic as LCS). Clearly, L-equivalence implies l-equivalence. The converse is true for k-spaces but not in general.

2. For every X the free Abelian group $A(X)$ is isomorphic to a subgroup of the unitary group. For free non-Abelian groups this remains open.

3. There exists a countable metrizable group G that is projectively universal: every countable metrizable group is isomorphic to a quotient of G (this is joint with Pestov). The completion of such a group is a projectively universal Polish group.